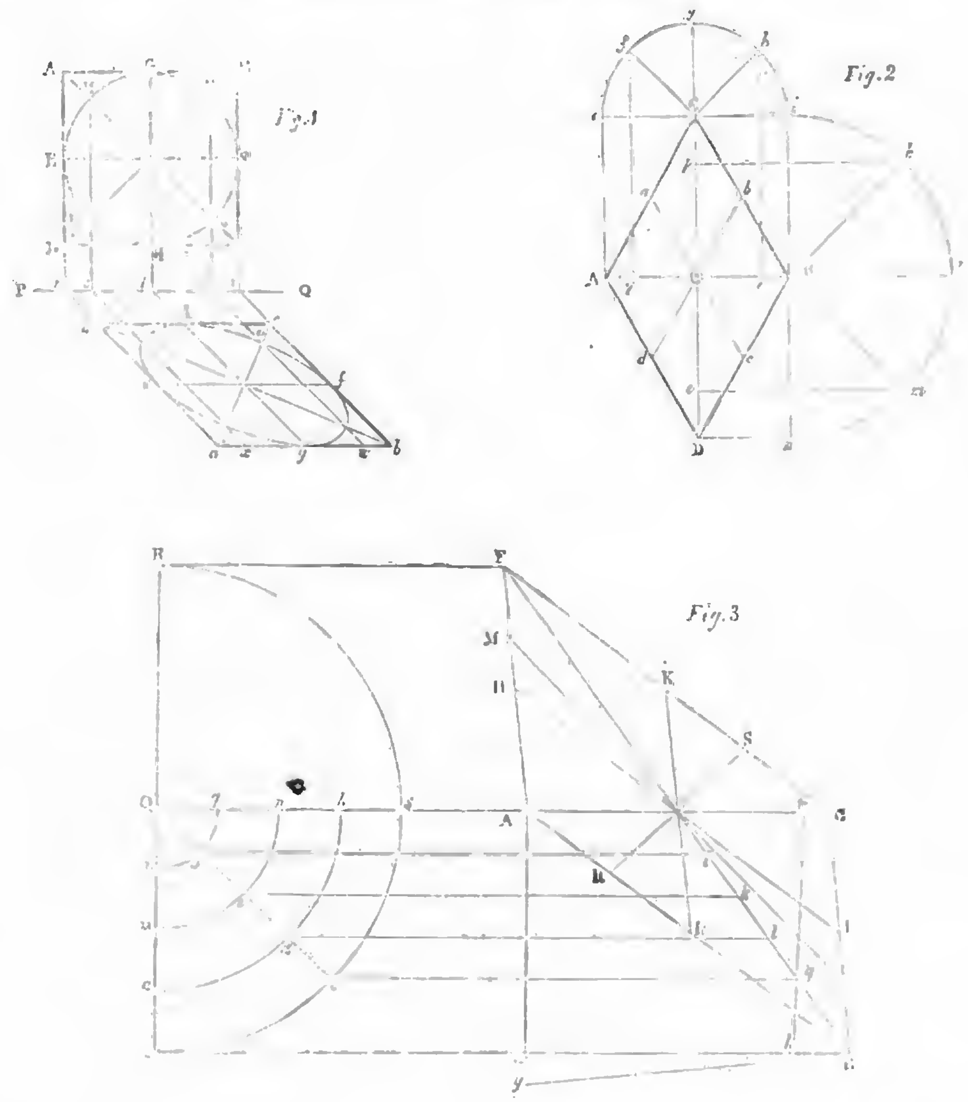
GRADATION OF THE ELLIPSES.



however, must be given, and the question is how. We would answer, throw open the nave by removing the screen at the west end of the choir. Objections have been raised to this, but all of them may be removed; and next week we will refer to them with that object in view. It is to be hoped that the dean and chapter will pause yet a little.

ON THE GRADATION OF ELLIPSES.

In architecture and some other branches of the constructive arts, it frequently happens that a number of concentric ellipses have to be delineated in the same plane, and in such a manner, that the space contained between any two contiguous boundaries shall not be of equal wideness all round, but foreshortened in one direction and elongated in the other. This is called the *gradation of ellipses*, and an easy and expeditious mode of effecting the gradation is a desideratum which the present paper is intended to supply.

If a circle and its circumscribing square be viewed obliquely in a given angle, the orthographic projection of the square will become a rhombus, and that of the circle an ellipsis; this is the first principle to be attended to in the theory which we are now about to explain, and since the principle is simple, the theory to which it is to be applied will be found to partake of a similar character.

Let ABCD in the annexed drawing, fig. 1, be the given square, and E G F H its inscribed circle, and let it be supposed that the rays are incident at an angle of forty-five degrees. Draw the diagonals A C and B D intersecting each other in ⊙ and the circumference of the circle in N, M, R, and S; and through the centre ⊙, and parallel to the sides of the square, draw the straight lines E F and G H meeting the vertical and horizontal sides in E, F and G, H; then will the projections of the four points in the sides of the square, with the four corresponding points in the circumference of the circle, be eight points in the circumference of the projected ellipsis; this at once points out the principle of construction for the projected figures.

Assume any point P in the plane of the figure, and through P draw the straight line PQ parallel to DC, the side of the square, and produce the vertical sides AD and BC to meet PQ in *r* and *v*, and through the points M, G, and N draw the vertical lines M*s*, G*t*, and N*u* to meet the same line PQ in *s*, *t*, and *u*. Then, at the several points *r*, *s*, *t*, *u*, and *v* make angles equal to forty-five degrees, so that the straight lines *ra*, *sx*, *tg*, *uw*, and *vb* may lie in the direction of the incident rays. On *ra* or any other of these parallels set off the distances *rd*, *re*, and *ra* respectively, equal to *r*D, *r*E, and *r*A, and through *d*, and *a* draw *dc*, and *ab* parallel to the directrix PQ and meeting *vb* in c and *b*; then is the rhombus *abcd* the projected representation of the square ABCD. Draw the diagonals *ac* and *bd*, and the parallel *ef*; then will the several points of intersection at *e*, *m*, *h*, *o*, *f*, *n*, *g*, and *p*, be eight points in the circumference of the projected ellipsis, through which points the curve itself may very easily be traced.

It may sometimes happen, but more especially in the projection of solid bodies, that the rhombus or parallelogram itself is given in which it is required to inscribe an ellipsis, or a series of concentric ellipses having the condition formerly alluded to; in this case the square and its inscribed circle by which we determined the points of projection are not given, but it is easy to perceive that by a reverse process, the square and the circle could very readily be found; it is, however, not necessary to do so, for by examining the preceding construction a method will instantly suggest itself of doing the same thing directly from the rhombus as it is given.

Let ACBD, fig. 2, be the given rhombus in which it is required to inscribe an ellipsis, by means of eight determinate points in its circumference. Draw the diagonals AB and CD, intersecting each other in the point ⊙, which is the centre of magnitude of the figure: then through ⊙, and parallel to the sides of the rhombus, draw the straight lines *ac* and *bd*, marking out the points *a*,*c* and *b*,*d*, which are four points in the circumference of the ellipsis, corresponding to *e*,*f* and *h*,*g* in the preceding construction.

Through A and B, the extremities of the diagonal AB, draw the perpendiculars A*e* and